

COMPETITION 9 - MATHEMATICS OF NATURAL SELECTION

Part 1:

$$P_{A_0} = 0.4$$

$$u = 0$$

$$P_{A_1} = \frac{P_{A_0}(1+u)}{1+uP_{A_0}(2-P_{A_0})}$$

$$= \frac{0.4(1+0)}{1+0 \times 0.4(2-0.4)}$$

$$= \frac{0.4}{1}$$

$$= 0.4$$

If $u=0$ then P_{A_t} remains the same, so is constant

$$P_{A_0} = P_{A_1} \therefore P_{A_t} = 0.4$$

$$P_{A_0} = 0.4$$

$$u = 0.4$$

$$P_{A_1} = \frac{P_{A_0}(1+u)}{1+uP_{A_0}(2-P_{A_0})}$$

$$= \frac{0.4(1+0.4)}{1+0.4 \times 0.4(2-0.4)}$$

$$= \frac{0.56}{1.256}$$

$$= \frac{70}{157} = 0.4458598726114 \dots$$

$$P_{A_2} = \frac{\frac{70}{157}(1+0.4)}{1+0.4 \times \frac{70}{157}(2-\frac{70}{157})}$$

$$= 0.4887392395$$

$$P_{A_3} = \frac{(0.48873\dots)(1+0.4)}{1+0.4 \times (0.48873\dots)(2-0.48873\dots)}$$

$$= 0.5281852561$$

$$P_{A_4} = \frac{(0.52818\dots)(1+0.4)}{1+0.4 \times (0.52818\dots)(2-0.52818\dots)}$$

$$= 0.5640610115$$

If $u=0.4$ then P_{A_t} increases non-linearly for the first 5 generations

$$P_{A_5} = \frac{(0.56406\dots)(1+0.4)}{1+0.4 \times (0.56406\dots)(2-0.56406\dots)}$$

$$= 0.5964468487$$

$$P_{A_0} = 0.4$$

$$u = -0.4$$

$$P_{A_1} = \frac{P_{A_0}(1+u)}{1+u \times P_{A_0}(2-P_{A_0})}$$

$$= \frac{0.4(1-0.4)}{1-0.4 \times 0.4(2-0.4)}$$

$$= \frac{10}{31} = 0.322580645162 \dots$$

$$P_{A_2} = \frac{\frac{10}{31}(1-0.4)}{1-0.4 \times \frac{10}{31}(2-\frac{10}{31})}$$

$$= \frac{62}{251} = 0.2470119521912 \dots$$

$$P_{A_3} = \frac{\frac{62}{251}(1-0.4)}{1-0.4 \times \frac{62}{251}(2-\frac{62}{251})}$$

$$= 0.1792547371$$

$$P_{A_4} = \frac{(0.1792 \dots)(1-0.4)}{1-0.4 \times (0.1792 \dots)(2-0.1792 \dots)}$$

$$= 0.1237022851$$

$$P_{A_5} = \frac{(0.1237 \dots)(1-0.4)}{1-0.4 \times (0.1237 \dots)(2-0.1237 \dots)}$$

$$= 0.08181737159$$

If $u = -0.4$ then P_{A_t} decreases non-linearly for the first 5 generations

Part 2:

• The green line remains constant at $P_{A_t} = 0.4$

• The ratio between the numerator and the denominator is greater in the equation for the blue line, meaning it's rate of increase is less than that of the red lines' decrease.