Part 1

$$P_{A_{t+1}} = \frac{P_{A_t}(1+V) - V}{1 + V P_{A_t}(2 - P_{A_t}^{0})}$$

If $P_{A_0} = 0.4$ $t = 1$ then $t = 5$

If $V = 0$ then 0.4 then $+0.4$

A.

$$P_{A_1} = \frac{0.4(1+0)}{1+(0)(0.4)(2-0.4)} = \frac{0.4}{1+0} = 0.4$$

$$P_{A_2} = \frac{P_{A_1}(1+V)}{1+V P_{A_1}(2-P_{A_1})} = \frac{0.4(1+0)}{1+(0)(0.4)(2-0.4)} = \frac{0.4}{1+0} = 0.4$$

∴ when $V = 0$ then the difference between $t=1$ and $t=5$ is 0

B.

$$P_{A_1} = \frac{0.4(1+0.4)}{1+(0.4)(0.4)(2-0.4)} = \frac{0.4 \times 1.4}{1+(0.16)(1.6)} = \frac{0.56}{1+0.256} = \frac{70}{157} = 0.44585\ldots$$

$$P_{A_2} = \frac{0.44\ldots(1+0.4)}{1+(0.4)(0.44\ldots)(2-0.44)} = \frac{0.44\ldots \times 1.4}{(1+(0.4 \times 0.44\ldots)(2-0.44)} = 0.4887\ldots$$

$$P_{A_{t+1}} = (\text{ans} \times 1.4) \div (1+(0.4 \times \text{ans})(2-\text{ans}))$$

$P_{A_3} = 0.5281\ldots$

$P_{A_4} = 0.56406\ldots$

$P_{A_5} = 0.596446848$

The difference between $t=1$ and $t=5$ is $0.150576475\ldots$

0.151

C

$$P_{A_{t+1}} = \frac{\text{ans}(1+(-0.4))}{1+(-0.4)(\text{ans})(2-\text{ans})} = \frac{0.6\,\text{ans}}{(1+(-0.4\,\text{ans})(2-\text{ans}))}$$

$P_{A_1} = 0.322580645$

$P_{A_2} = 0.247\ldots$

$P_{A_3} = 0.179\ldots$

$P_{A_4} = 0.123\ldots$

$P_{A_5} = 0.008181737 1$

The difference between $t=1$ and $t=5$ is -0.240763273

-0.241

Part 2

The green line stays at $P_{A_t} = 0.4$ because the equation when $v = 0$ simplifies to $P_{A_{t+1}} = P_{A_t}$ so P_{A_t} will never change creating a flat line on the graph

The red line ($v = -0.4$) goes to 0 faster than the blue line ($v = 0.4$) goes to 1 because

The red line

$$P_{A_{t+1}} = \frac{0.6 P_{A_t}}{1 - 0.8 P_{A_t} + 0.4 P_{A_t}^2}$$

$0.6 P_{At}$ means the numerator decreases by 40%.

Because the value of P_{At} will always be between 0 and 1 as it is probability P_{At}^2 will be smaller than P_{At}

so $-0.8 P_{At} + 0.4 P_{At}^2$ is always negative.

The denominator is less than one so the equation will be slightly more than $0.6 P_{At}$

However as t increases and P_{At} decreases the denominator will increase each time making the next value closer to 0.6 of the previous value

t	1	2	3
P_{At}	$\frac{0.24}{0.744}$	$\frac{0.193}{0.783}$	$\frac{0.148}{0.826}$

The blue line

$$P_{A_{t+1}} = \frac{1.4 P_{A_t}}{1 + 0.8 P_{A_t} - 0.4 P_{A_t}^2}$$

$1.4 P_{At}$ means the numerator increases by 40%.

Because the value of P_{At} will always be between 0 and 1 as it is probability P_{At}^2 will be smaller than P_{At}

so $+0.8 P_{At} - 0.4 P_{At}^2$ is always positive

The denominator is more than 1 so the equation will be slightly less than $1.4 P_{At}$

As t increases and P_{At} increases the denominator will increase each time making the next value even less than 1.4 of the previous value

t	1	2	3
P_{At}	$\frac{0.56}{1.256}$	$\frac{0.624}{1.277}$	$\frac{0.684}{1.295}$

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