St John's College Inspire Programme → Competition 8

Key points

→ The disease shows an autosomal recessive inheritance.

→ A person must be heterozygous for this trait in order to pass on the disease (since being homozygous with the recessive allele would prevent one from having children).

→ We are not considering genetic demographic factors - the recessive allele may be more or less prevalent within certain communities that may be more likely to co-parent within said community.

→ The dominant allele ~~with~~ will hereafter be referred to as B, while the recessive allele will be referred to as A.

Part 1) Both parents are heterozygous, ie., AB.

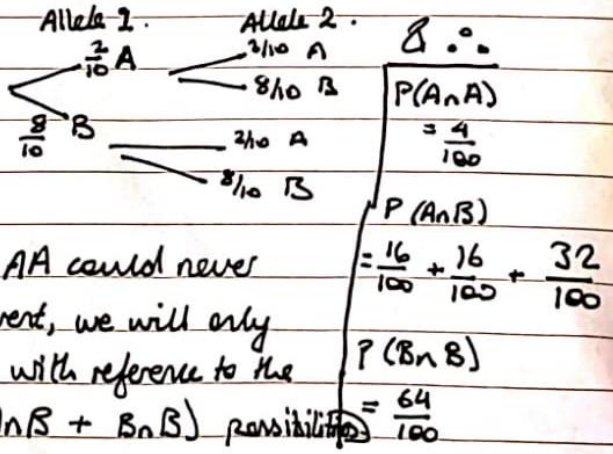
	A	B
A	AA	AB
B	AB	BB

Therefore, there is a $\frac{1}{4}$ or 25% of inheriting the disease (and an additional 50% chance of being a carrier).

Part 2) Assuming here that 'prevalence' refers to the ratio of the number of a specified allele in a population to the total of all alleles at its genetic locus:

P_1 (parent 1) is AB.

P_2 (parent 2)



Since AA could never be a parent, we will only consider with reference to the other (A∩B + B∩B) possibilities.

∴ $P(\text{parent A}\cap B \mid \text{parent}) = \frac{32}{96}$

& $P(B\cap B \mid \text{parent}) = \frac{64}{96}$

If the P_2 was AB, the chances of the child being a sufferer would be:

	A	B
A	AA	AB
B	AB	BB

$= \frac{1}{4}$

The overall chance of this is $\frac{32}{96} \times \frac{1}{4} = \frac{8}{96}$. (weighted)

If P_2 was BB, the chances of the child being a sufferer would be:

	B	B
A	AB	AB
B	BB	BB

$= 0$

This would happen in $\frac{0}{96}$ cases (NEVER).

So the overall chance of the child being AA is $\frac{8}{96}$ or $\frac{1}{12}$.

Part 3) Selecting two people at random (who are capable of being parents):

P_2 can either be AB $\rightarrow \frac{32}{96}$ (as can P_2)
or BB $\rightarrow \frac{64}{96}$, (R)

The possible pairings are $P_2(R)\ P_2(R) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ chance

Where:
$P_x(T)$ refers to AB
and $P_x(R)$ refers to BB.

- $P_2(T)\ P_2(T) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ chance
- $P_1(R)\ P_2(T) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ chance
- $P_1(T)\ P_2(R) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ chance

$P_1(R)\ P_2(R)$ → no chance of child developing or carrying.

$P_1(T)\ P_2(T)$

	A	B
A	AA	AB
B	AB	BB

→ $\frac{1}{4}$ chance, which is $\frac{1}{4} \times \frac{1}{9} = \frac{1}{36}$ when weighted.

$P_1(R)\ P_2(T)$ (which is same as chances $P_1(T)\ P_2(R)$)

	B	B
A	AB	AB
B	BB	BB

→ No chance of carrying.

So, the total chance is $\frac{1}{36}$.

Part 4) One parent is known to be AB.

We know that second parent cannot be AA, and therefore, as above proven, there is a (within the general population) $\frac{1}{3}$ chance of AB, $\frac{2}{3}$ chance of BB.

If P_2 is AB:

	A	B
A	AA	AB
B	AB	BB

→ $\frac{1}{2}$ chance of child being a carrier,

⇒ weighted chance of $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

If P_2 is BB:

	B	B
A	AB	AB
B	BB	BB

→ $\frac{1}{2}$ chance

⇒ weighted chance $= \frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$

Thus, the net chance is $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$ OR $\frac{1}{2}$.

Part 5) I am AB (or BA), so it has just been proven that my child has a $\frac{1}{2}$ chance of also ~~suffering from~~ being a carrier of the disease.

As another AB, his children will, in turn, have a $\frac{1}{2}$ chance of being carriers.

∴ The overall chance is $\frac{1}{4}$.