$\lim_{war \to \infty} Avengers = \frac{1}{2}$

LIMITS AND L'HOPITAL'S RULE

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Definition of a limit

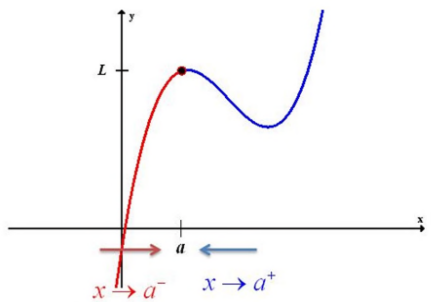
$$\lim_{x \to a} f(x) = L$$

We make the values of *f(x)* arbitrarily close to L by taking x to be sufficiently close to *a* (on either side of *a*) but not equal to *a*.
This states that the values of *f(x)* tend to get closer and closer to a number *L* as *x* gets closer to the number *a* (from the left and right side of *a*) but $x \neq a$. When finding limits, we never consider $x=a$. In fact, it is not necessary for *f(x)* to even be defined at $x=a$. It only matters how *f* is defined near *a*.

Limits only exist when…

A limit only exists if the left hand limit of *f(x)* = the right hand limit of *f(x)*. We can write:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$



The epsilon & delta definition

We wish to express that *f(x)* can be made to be arbitrarily close to *L* when making *x* close to a ($x \neq a$). Therefore, *f(x)* can be made to lie any preassigned distance from L, ε, by requiring that *x* be within a specified distance δ from *a*. We can write:

$$|f(x) - L| < \varepsilon \text{ when } |x - a| < \delta$$

Thus the following definition:

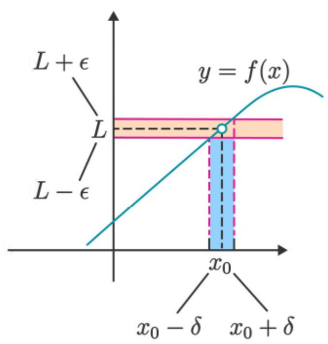
If you let *f* be a function defined on some open domain that contains the number *a*, then

$$\lim_{x \to a} f(x) = L$$

if for every number

$\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

$$0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon$$



Limit Rules

Suppose that f(x) has limit L and g(x) has limit M, then:

- $\lim_{x \to a} kf(x) = kL$
- $\lim_{x \to a} \big(f(x) \pm g(x)\big) = L \pm M$
- $\lim_{x \to a} \big(f(x) \cdot g(x)\big) = L \cdot M$
- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$
- $\lim_{x \to a} \big(f(x)^k\big) = L^k$

L'Hopital's Rule

If when computing a limit, you try to cancel factors (supposing they are not 0) or multiplying by the conjugate and it still returns an indeterminate form such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$, we can introduce a systematic method to evaluate indeterminate forms called L'Hopital's Rule.

Suppose *f* and *g* are differentiable and $g'(x) \neq 0$, near *a*, then:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Only if:

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ or
If $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$