**Energy recovery from the tides**

Each day, as the moon moves around the earth, the ocean waters rise and fall. In deep oceans the height of rise and fall (called the tidal range) may be only part of a metre, but near coasts the tidal range is much larger, usually a few metres, and on some coasts, can be much larger, 8-10m and even nearer 15m in a few places, for example, the Bay of Fundy in Canada. Here we will look at some of the quantities that relate to whether it is feasible to recover energy from the tides.

If we move a mass $m$ in kilograms up a height $h$ in metres against gravity $g$ then the energy gain of the mass, $E$, in Joules, is

$$E = mgh.$$

Suppose for the moment that we construct in the sea in a coastal region a basin of area $A$ that could fill and empty as the sea height rose and fell with a tidal range $H$. The energy change for either rise or fall of the sea for water of density $\rho$ in the basin is

$$E = \frac{1}{2}\rho g A H^2$$

**Question:** Why is there a factor of a half in this formula?

**Answer:** *If you like to think with physical understanding, then as the basin fills, water that first comes in will gain very little potential energy, whereas water that comes in when the basin is nearly filled will gain potential energy for rising a height that approaches* $H$ *as the basin completes finishing, and the average rise in potential energy will be the same as a mass* $\rho A H$ *rising a height* $H/2$ *against gravity* $g$*. If you like to think mathematically, then a horizontal layer of thickness* $\mathrm{d}h$ *that has risen a height* $h$ *will gain potential energy* $\rho g A h \mathrm{d}h$ *and the overall gain will be*

$$\int_0^H \rho g A h \mathrm{d}h = \frac{1}{2}\rho g A H^2$$

If energy could be harvested for both rise and fall and occured over a time $T$ (for tides, this is slightly more than one day, but assume it is one day, $T = 24 \times 60 \times 60 =$ seconds), then the average power generated, in Watts would be,

$$P = \frac{E}{T}.$$

**Question:** Calculate the theoretical maximum average power produced for a square basin of side 100m with tidal range 5m. Of course the actual power would be much less, most likely around only 25% of the theoretical maximum, the ratio of actual to theoretical power is called a capacity factor. Assume the density of sea water is $\rho = 1030\mathrm{kg/m^3}$.

**Answer:** *Substituing into the formulae we have above,*

$$E = 0.5 \times 1030 \times 9.81 \times 100^2 \times 5 = 252607500 \text{ Joules},$$

*with the period*

$$T = 86400 \text{ seconds},$$

*and so the average power output would be*

$$P = \frac{252607500}{86400} \approx 2924 \text{ Watts}.$$

**Question:** What area of basin would be needed for the actual average power output to be 100MW, assuming a capacity factor of 25% and a tidal range of 8m?

**Answer:** *To determine the area we need to re-arrange the formula for power and include a capacity factor, let's denote this factor $f$, then we obtain*

$$A = \frac{2ET}{f \rho g H^2}.$$

*For this example*

$$A = \frac{2 \times 10^8 \times 86400}{0.25 \times 1030 \times 9.81 \times 8^2} \approx 107 \text{ km}^2,$$

*so a basin around* 10 *km on each side, making site location very problematic, requiring a huge construction cost and having potential environmental damage over a large area. The natural conclusion is to look for relatively large coastal inlets or bays with good tidal range where the coast itself provides most of the 'boundary' of the basin and construction is only across a 'neck' of water, such as the Rance site in France.*

If you are interested in learning more about tidal power, you might look at
https://en.wikipedia.org/wiki/Rance_Tidal_Power_Station
or just search online for tidal power. There are many reasons why tidal power may be difficult to harvest, notably high cost of construction and long payback time, and adverse environmental consequences, but it is undeniable that the tides will continue and the sea level rise and fall!