Economics Problem: Magic Money Tree

1. Suppose you start with £4 as in the example, so you plant £4.60 for Year 2. How much new money will you grow in Year 2? How much will you spend? How much money will you plant in total at the end of Year 2? What happens in year 3? How is the amount of money that you have changing over time?

In Year 2 you will grow £2√(4.6) = £4.29

You will spend 0.75 × £4.29 = £3.22 and save £1.07 for planting

The mice will eat 0.1×£4.6 = £0.46

Therefore you will plant a total of £4.60 -£0.46+£1.07 = £5.21 for Year 3

The following table shows what happens in each year for the first 10 years. We can see that the amount of money that you have each year keeps growing, but at a decreasing rate.

year	plant	grow	spend	mice
1	4.00	4.00	3.00	0.40
2	4.60	4.29	3.22	0.46
3	5.21	4.57	3.42	0.52
4	5.83	4.83	3.62	0.58
5	6.46	5.08	3.81	0.65
6	7.08	5.32	3.99	0.71
7	7.70	5.55	4.16	0.77
8	8.32	5.77	4.33	0.83
9	8.93	5.98	4.48	0.89
10	9.53	6.18	4.63	0.95

2. What would happen if you started with £100 instead of £4?

If you start at £100, then the amount that you plant and spend each year would instead keep decreasing, but again this would slow down over time. The table below shows what happens for the first 10 years.

year	plant	grow	spend	mice
1	100.00	20.00	15.00	10.00
2	95.00	19.49	14.62	9.50
3	90.37	19.01	14.26	9.04
4	86.09	18.56	13.92	8.61
5	82.12	18.12	13.59	8.21
6	78.44	17.71	13.28	7.84
7	75.02	17.32	12.99	7.50
8	71.85	16.95	12.71	7.19
9	68.90	16.60	12.45	6.89
10	66.16	16.27	12.20	6.62

3. If you keep repeating the process of growing, spending and planting money, what happens in the long run to the amount of money that you have? Does it depend on how much you had to start with?

Looking at what happens when you start at £100 or £4, it looks as if the amount of money you plant is getting closer and closer to some fixed amount somewhere between £9.53 and £66.16. When you have a small amount such as £4, the amount that you save each year is bigger than the amount the mice eat, so your total money grows. When you have a large amount such as £100, the amount that you save each year is smaller than the amount the mice eat, so your total money falls. We can find the amount of money X at which the money you have will stay exactly the same by solving the following equation, where the amount of money saved just equals the amount eaten by mice.*

$$(2\sqrt{X^*})/4 = X^*/10$$
$$5\sqrt{X^*}=X^*$$
$$\sqrt{X^*}=5$$
$$X^*=25$$

If you have less than £25, then the money you have will increase each year. If you have more than £25, then the money you have will decrease each year. In each case the amount you have will get closer and closer to 25 (though never exactly reaches it).

4. What would happen if you changed the fraction of new money that you plant from one quarter to something else?

If you increase the fraction that you save for planting, then the money you have will grow faster (or shrink more slowly). Your money will converge to a higher amount X. If s is the fraction of new money that you save for planting, then X* is given by*

$$s(2\sqrt{X^*})= X^*/10$$
$$20s\sqrt{X^*}=X^*$$
$$\sqrt{X^*}=20s$$
$$X^*=400s^2$$

which is increasing in s.

5. Suppose that the only thing you care about is the amount of money that you get to spend each year in the long run. What fraction of your new money should you plant each year?

When you choose your saving (planting) rate s, you face a trade-off. On the one hand a higher s leads to a higher long run level of wealth X^**, but you spend a smaller proportion of it. The level of long run spending is given by*

$$F(s) = (1-s)400s^2$$

To find the saving rate s that maximizes F(s) we need to find and solve the first order condition:*

$$\frac{dF}{ds} = 400(2s-3s^2) = 0 \text{ at a maximum}$$

$$s(2-3s) = 0$$
$$s = 0 \text{ or } s = \frac{2}{3}$$

The solution s=0 is a minimum point, not a maximum (it would lead to zero wealth and spending in the long run). So the planting rate that would maximise long run saving is s = 2/3.*